EMPTYING A PIPE CONTAINING A FLUID WITH A CHANGING STATE OF AGGREGATION

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A solution is obtained for the dynamic problem of emptying a pipe closed at one end and containing a fluid nonuniformly heated along its length, for the case in which the saturated vapor pressure of the hottest layers exceeds the ambient pressure. The solutions in quadratures in terms of the dimensionless quantities are approximated by formulas.

In studying the dynamic processes taking place in certain heat engines, it is necessary to solve the following problem. Under transient conditions, the hydraulic header through which the coolant moves is shut off, and its inlet is connected to the ambient medium. As a rule, since the atmospheric pressure (usually a low vacuum) is less than the saturated vapor pressure of the hottest layers of fluid adjacent to the cutoff valve, these layers begin to boil. The vapor-liquid emulsion thus formed expands, displacing the nonboiling layers of fluid from the pipe (the fluid moves in a direction opposite to its normal direction of motion). With time, the temperature of the emulsion and, hence, the vapor pressure fall, which leads to the boiling of new layers of liquid.

Our object is to determine the rate at which the pipe fills with boiling liquid and to find the time dependences of the displacement pressure and the boiling rate. We make the following assumptions: a) there is no heat exchange with the pipe walls; b) the vapor-liquid emulsion is uniformly dispersed with parameters constant over the entire volume; c) phase transitions are instantaneous (without a time lag); i.e., the process is assumed to be in thermodynamic equilibrium; d) the temperature of the liquid does not exceed the critical point; e) in the outlet section the liquid is not able to start boiling; f) the temperature distribution in the liquid is linear along the length of the pipe; g) friction losses are negligibly small.

We write the principal equations describing this process.

The energy equation for the boiling liquid can be written in the form [3]

$$\frac{d(um)}{d\tau} = -\frac{dL}{d\tau} + i'G. \tag{1}$$

The change of mass of the vapor-liquid emulsion per unit time is equal to the boiling rate per second

$$\frac{dm}{d\tau} = G. \tag{2}$$

The work done by the emulsion is equal to

$$\frac{dL}{d\tau} = p\,\frac{dV}{d\tau}. \tag{3}$$

The rate at which the pipe is filled with boiling liquid is given by

$$\frac{dV}{d\tau} = v'(G' + G), \tag{4}$$

where

$$G' = \chi\sqrt{p - p_a} \tag{5}$$

and

$$\chi = \mu F \sqrt{\frac{2}{\xi v'}}.$$

The temperature distribution along the length of the pipe obeys the law

$$T' = T_0 - kV'.$$

Since the motion of the liquid does not affect its temperature, the mass of the vapor-liquid emulsion is determined by the initial temperature T_0 of the hottest layers of liquid and the variable emulsion temperature T, i.e.,

$$m = \frac{T_0 - T}{kv'}, \tag{6}$$

and the volume of the pipe filled with boiling liquid is

$$V = \frac{v(T_0 - T)}{kv'}. \tag{7}$$

The relation between the parameters of the emulsion, the liquid, and the vapor is given by the well-known equations of thermodynamics [4]

$$u = i - pv, \tag{8}$$

$$v = v' + \psi(v'' - v'), \tag{9}$$

$$i = i' + \psi r, \tag{10}$$

and the equation of state for a two-phase fluid in process of changing its state of aggregation has the form [2]

$$p = p(T). \tag{11}$$

With sufficient accuracy, the physical parameters of the liquid and vapor—enthalpy, specific volume, specific heat, heat of evaporation—can be assumed to be functions of pressure or temperature only [1, 5]. Represented in terms of the dimensionless pressure $\bar{p} = p/p_{cr}$ or dimensionless temperature $\bar{T} = T/T_{cr}$, for thermodynamically similar substances these functions of each parameter differ by a constant multiplier; i.e.,

$$v' = v_* \pi_v', \qquad (12)$$

$$v'' = v_* \pi_v'', \qquad (13)$$

$$i' = i_* \pi_i', \qquad (14)$$

$$r = r_* \pi_r, \qquad (15)$$

$$c_p' = c_*' \pi_c, \qquad (16)$$

where graphs of the universal functions π_χ and values of the constant coefficients χ_* for certain cases can be found in the works of V. M. Borishanskii and P. I. Povarnin.

The written system of equations contains unknown quantities, which can be represented as functions of the pressure $\bar{p}$ or temperature $\bar{T}$. Therefore, having determined the law of variation of $\bar{p}$ or $\bar{T}$ with time, we can also find the variation of the other parameters of interest.

We first determine the dependence of the vapor content ψ on the dimensionless pressure $\bar{p}$. For this purpose, we transform Eq. (1) to the linear differential equation

$$\frac{d\psi}{d\bar{p}} + \pi_1\psi = \pi_2, \qquad (17)$$

where

$$\pi_1 = \frac{d \ln \pi_r}{d\bar{p}} - \frac{p_{cr} v_*}{r_*} \frac{(\pi_v'' - \pi_v')}{\pi_r} - \frac{1}{\bar{T}_0 - \bar{T}} \frac{d\bar{T}}{d\bar{p}};$$

$$\pi_2 = \frac{1}{r_* \pi_r}\left(p_{cr} v_* \pi_v' - i_* \frac{d\pi_i'}{d\bar{p}} \right).$$

We integrate Eq. (17) from $\bar{p}_0$ to $\bar{p}$ and from 0 to ψ:

$$\psi = \exp\left(-\int_{\bar{p}_0}^{\bar{p}} \pi_1 d\bar{p}\right) \int_{\bar{p}_0}^{\bar{p}} \pi_2 \exp\left(\int_{\bar{p}_0}^{\bar{p}} \pi_1 d\bar{p}\right) d\bar{p}. \qquad (18)$$

Using this equation we can also find the relation between $\bar{p}$ and τ. For this purpose we differentiate (7) and, substituting the value obtained for the derivative in (4), using (5) and (12)–(17) we obtain

$$d\tau = \frac{T_{cr}}{k\mu F} \sqrt{\frac{\xi}{2p_{cr}v'}} P(\bar{p}) d\bar{p}, \qquad (19)$$

where

$$P(\bar{p}) = (\bar{p} - \bar{p}_a)^{-\frac{1}{2}} \left\{ (\bar{T}_0 - \bar{T}) \pi_2 \left(\frac{\pi_v''}{\pi_v'} - 1 \right) + \psi \left[(\bar{T}_0 - \bar{T}) \left(\frac{d\left(\frac{\pi_v''}{\pi_v'}\right)}{d\bar{p}} - \pi_1 \left(\frac{\pi_v''}{\pi_v'} - 1 \right) \right) - \left(\frac{\pi_v''}{\pi_v'} - 1 \right) \frac{d\bar{T}}{d\bar{p}} \right] \right\}.$$

We integrate (19), going over to the dimensionless time

$$\bar{\tau} = \int_{\bar{p}_0}^{\bar{p}} P(\bar{p}) d\bar{p}, \qquad (20)$$

where

$$\bar{\tau} = \tau/\tau_*, \quad \tau_* = \frac{T_{cr}}{k\mu F} \sqrt{\frac{\xi}{2p_{cr}v'}}.$$

The integral obtained depends only on the dimensionless pressure and can therefore be found by a numerical method.

The results of numerical integration can be approximated by a formula of the type

$$\lg \bar{p} = a \exp(b \lg \bar{\tau}), \qquad (21)$$

where

$$a = a_1 \bar{T}_0 + a_2, \quad b = b_1 \bar{T}_0 + b_2.$$

For water: $a_1 = 4.05$; $a_2 = -4.18$; $b_1 = 1.44$; $b_2 = 0.874$.

Knowing the dependence of $\bar{p}$ on $\bar{\tau}$, using (5) we can easily determine G'.

The volume of boiled liquid can be found using Eq. (7):

$$\bar{V} = (\bar{T}_0 - \bar{T}) \left[1 + \psi \left(\frac{\pi_v''}{\pi_v'} - 1 \right) \right],$$

where

$$\bar{V} = V/V_*, \quad V_* = T_{cr}/k.$$

The time required for the entire pipe to fill with boiling liquid is found from the condition $V = V_{pipe}$.

After the pipe has filled with vapor-liquid emulsion, the problem becomes more complicated, since the rate of flow of a two-phase fluid which is changing its state of aggregation cannot be found from Eq. (5). Accordingly, this problem requires separate consideration.

Thus, the formulas obtained make it possible to find the parameters of a fluid undergoing a change of state of aggregation during the emptying of a pipe, which simplifies the numerical calculation of the transient regimes of heat engines.

NOTATION

c_p is the specific heat; G is the mass boiling rate per second; i is the enthalpy; k is some constant coefficient; L is the work; F is the clear cross section; p is the pressure; r is the heat of evaporation; m is the mass; T is the temperature; u is the internal energy; V is the volume; v is the specific volume; ξ is the loss factor; μ is the flow coefficient; τ is the time; ψ is the vapor content. Subscripts: single prime—liquid; double prime—vapor; *—constants which depend on the nature of the liquid; a—atmospheric conditions; 0—initial conditions.

REFERENCES

1. V. M. Borishanskii, collection: Problems of the Heat Transfer and Hydraulics of Two-Phase Media [in Russian], Gosenergoizdat, 1961.

2. G. L. Grodzovskii, Yu. E. Kuznetsov, and G. E. Khudyakov, Trudy TsKTI, no. 59, 1965.

3. M. A. Mamontov, Problems of the Thermodynamics of a Body of Variable Mass [in Russian], Oborongiz, 1961.

4. N. I. Novikov and K. D. Voskresenskii, Applied Thermodynamics and Heat Transfer [in Russian], Gosatomizdat, 1961.

5. P. I. Povarnin, collection: Heat Transfer [in Russian], Izd. AN SSSR, 1962.

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